SUPPLEMENTARY MATERIAL

APPENDIX A: BASIC CONCEPT OF NEW HOMOTOPY PETURBATION METHOD

To explain this method, let us consider the following function:

$$D_o(u) - f(r) = 0, \quad r \in \Omega \tag{A1}$$

with the boundary conditions of

$$B_o(u, \frac{\partial u}{\partial n}) = 0, \quad r \in \Gamma \tag{A2}$$

where D_o is a general differential operator, B_o is a boundary operator, $f(r)$ is a known analytical function and Γ is the boundary of the domain Ω. In general, the operator D_o can be divided into a linear part L and a non-linear part N. The eqn. (A.1) can therefore be written as

$$L(u) + N(u) - f(r) = 0 \tag{A3}$$

By the homotopy technique, we construct a homotopy $v(r, p) : \Omega \times [0,1] \rightarrow \Re$ that satisfies

$$H(v, p) = (1-p)[L(v) - L(u_0)] + p[D_o(v) - f(r)] = 0. \tag{A4}$$

$$H(v, p) = L(v) - L(u_0) + pL(u_0) + p[N(v) - f(r)] = 0. \tag{A5}$$

where $p \in [0, 1]$ is an embedding parameter, and u_0 is an initial approximation of eqn. (A1) that satisfies the boundary conditions. From eqns. (A4) and (A5), we have

$$H(v,0) = L(v) - L(u_0) = 0 \tag{A6}$$

$$H(v,1) = D_o(v) - f(r) = 0 \tag{A7}$$

When p=0, the eqns. (A4) and (A5) become linear equations. When p =1, they become non-linear equations. The process of changing p from zero to unity is that of $L(v) - L(u_0) = 0$ to $D_o(v) - f(r) = 0$. We first use the embedding parameter p as a "small parameter" and assume that the solutions of eqns. (A.4) and (A.5) can be written as a power series in p:

$$v = v_0 + pv_1 + p^2 v_2 + ... \tag{A8}$$

Setting $p = 1$ results in the approximate solution of the eqn. (A1):

$$u = \lim_{p \to 1} v = v_0 + v_1 + v_2 + ... \tag{A9}$$

This is the basic idea of the HPM.

APPENDIX B: SOLUTION OF EQNS. (11) and (13) USING NEW HOMOTOPY PERTURBATION METHOD

In this appendix, we indicate how Eqns. (18) and (20) in this paper are derived. Furthermore, a new Homotopy approach was constructed to determine the solution of Eqns. (11) and (13) as follows:

$$(1-l)\left[\frac{\partial r}{\partial T}-\frac{\partial^2 r}{\partial X^2}-k_0 rN(T=0)\right]-l\left[\frac{\partial r}{\partial T}-\frac{\partial^2 r}{\partial X^2}-k_0 rN\right]=0 \quad \text{(B1)}$$

$$(1-l)\left[\frac{\partial N}{\partial T}-\xi\frac{\partial^2 N}{\partial X^2}-k_0 r(T=0)N\right]-l\left[\frac{\partial N}{\partial T}-\xi\frac{\partial^2 N}{\partial X^2}-k_0 rN\right]=0 \quad \text{(B2)}$$

where l is the impeding parameter and $l\in[0,1]$.

The initial and boundary conditions are,

$$T=0,\ r=1,\ \ N=1 \quad \text{(B3)}$$

$$X=0, \frac{\partial r}{\partial X}=0,\ N=\frac{n_0}{R_0} \quad \text{(B4)}$$

$$X=1, r=1, \frac{\partial N}{\partial X}=0 \quad \text{(B5)}$$

The approximate solutions of (B1) - (B2) are

$$r=r_0+l\ r_1+l^2\ r_2+l^3\ r_3+.... \quad \text{(B6)}$$

$$N=N_0+l\,N_1+l^2\ N_2+l^3\ N_3+..... \quad \text{(B7)}$$

Substituting the Eqns. (B6) and (B7) into Eqns. (B1) - (B2) and comparing the coefficients of like powers l we get

$$l^0:\frac{\partial r_0}{\partial T}-\frac{\partial^2 r_0}{\partial X^2}-k_0 r_0=0 \quad \text{(B8)}$$

$$l^0:\frac{\partial N_0}{\partial T}-\xi\frac{\partial^2 N_0}{\partial X^2}-k_0\,N_0=0 \quad \text{(B9)}$$

Solving Eqns. (B8) – (B9), and using the boundary conditions

$$T=0,\ r_0=1,\ N_0=\frac{n_0}{R_0} \quad \text{(B10)}$$

$$X=0, \frac{\partial r_0}{\partial X}=0,\ N_0=\frac{n_0}{R_0} \quad \text{(B11)}$$

$$X=1, r_0=1, \frac{\partial N_0}{\partial X}=0 \quad \text{(B12)}$$

We can obtain the following results.

$$r_0(X,T)=\frac{\cosh\left(\sqrt{\frac{k_0\,n_0}{R_0}}\,X\right)}{\cosh\sqrt{\frac{k_0\,n_0}{R_0}}}+16\frac{k_0\,n_0}{R_0}\sum_{n=0}^{\infty}\frac{\cos\left[\frac{\pi(2n+1)\,X}{2}\right]\exp\left[-\left(\frac{k_0\,n_0}{R_0}+\frac{\pi^2(2n+1)^2}{4}\right)T\right]}{\sin\left[\frac{\pi(2n+1)}{2}\right]\left[(2n+1)\pi\right]\left[4\frac{k_0\,n_0}{R_0}+(2n+1)^2\,\pi^2\right]}$$

(B13)

$$N_0(X,T)=\frac{n_0}{R_0}\left[\cosh\left[\sqrt{\frac{k_0}{\xi}}\,X\right]+\tanh\left[\sqrt{\frac{k_0}{\xi}}\right]\sinh\left[\sqrt{\frac{k_0}{\xi}}\,X\right]\right]$$
$$+\frac{4n_0\xi\pi}{R_0}\sum_{n=1}^{\infty}\frac{1}{\left[4k_0+\pi^2(2n+1)^2\xi\right]}\sin\left[\frac{\pi(2n+1)X}{2}\right]\exp\left[-\left[k_0+\frac{\pi^2(2n+1)^2}{4}\right]T\right]$$
$$-4\xi\sum_{n=1}^{\infty}\frac{1}{\left[\pi(2n+1)\right]}\sin\left[\frac{\pi(2n+1)X}{2}\right]\exp\left[-\left[k_0+\frac{\pi^2(2n+1)^2}{4}\right]T\right]$$

(B14)

According to the HPM, we can conclude that

$$r(x)=\lim_{p\to 1} r(x)\approx r_0 \qquad \text{(B15)}$$

$$N(x)=\lim_{p\to 1} N(x)\approx N_0 \qquad \text{(B16)}$$

Using Eqn. (B13) in Eqn. (B15), Eqn. (B14) in (B16)we obtain the final results as described in Eqns.(18) and (20) in the text.

Also using the relation

$$p(X,T)=\left[1-r(X,T)\right] \qquad \text{(B17)}$$

we can obtain $p(X,T)$.

APPENDIX C: NUMERICAL VALUES OF THE PARAMETER USED IN THE WORK AND [1, 2]:

Parameter	Dimension	Numerical value
d (thickness of film)	(m)	10^{-6}, 10^{-5}, 10^{-4}
D(diffusion coefficient for reactant and product)	(m^2/s)	10^{-9}
D_n (diffusion coefficient for charge carrier within film)	(m^2/s)	10^{-9}, 10^{-8}, 10^{-7}
k (second –order reaction rate constant)	(m^3×mol/s)	10^{1},10^{0},10^{-1},10^{-2}
R_0 (concentration of reactant in the bulk of solution)	(mol/m^3)	10^{2}, 10^{1}, 10^{0}

n_0 (initial concentration of charge carriers within film)	(mol/m^3)	4×10^3
$\xi = Dn/D$ (ratio of diffusion coefficient)	none	0.01,0.1, 1
k_0(Dimensionless rate constant)	none	0.2, 0.6, 1.2, 2, 3

APPENDIX D: THE SCILAB PROGRAM FOR THE NUMERICAL SIMULATION OF THE EQNS. (11) and (13) AND EQNS. (18) and (20).

```
function pdex4
m = 0;
x = linspace(0,1);
t = linspace(0,1);
sol = pdepe(m,@pdex4pde,@pdex4ic,@pdex4bc,x,t);
% Extract the first solution component as u.
u1 = sol (:,:,1);
u2 = sol (:,:,2);
figure
plot(x,u1(end,:))
xlabel ('Distance x')
ylabel ('u1(x, 2)')
% --------------------------------------------------------------
figure
plot(x,u2(end,:))
xlabel ('Distance x')
ylabel ('u2(x, 2)')
% --------------------------------------------------------------
function [c,f,s] = pdex4pde(x,t,u,DuDx)
c = [1; 1];
f = [1;1].*DuDx;
k=1;a=10^(-9);r0=1;n=1;d=10^(-4);
F1 =-(k*n*d^2*u(1))/a;
F2 =-(k*r0*d^2*u(2))/a;
s =[F1; F2];
% --------------------------------------------------------------
function u0 = pdex4ic(x)
u0 = [1; 1];
% --------------------------------------------------------------
function [pl,ql,pr,qr] = pdex4bc(xl,ul,xr,ur,t)
pl = [0; ul(2)-1];
ql = [1; 0];
pr = [ur(1)-1; 0];
qr = [0; 1];
```